

Students' Justifications for the Temporal Order of Delta and Epsilon in the Formal Definition of a Limit

Research that focuses on students' sense making about the formal definition of a limit, or the ε - δ definition, emphasizes the *temporal order* of delta and epsilon as a critical but challenging conceptual aspect of the definition. While some case studies have documented this difficulty, its generalizability and details of students' reasoning are underexplored. Semi-structured interviews with 18 students investigated students' reasoning through their justifications for the temporal order. diSessa's *Knowledge in Pieces* framework guided the analysis to explore the nature of students' claims, and relevant resources from students' prior knowledge and elsewhere. The study found that while many students shared the documented difficulty, most of them were able to identify the appropriate temporal order in at least one context. Students' understanding of functional dependence and logical implication, along with students' assertion that the limit in the definition was unknown, were ideas with *high priority* that contributed to the prevalence of the documented *x-first* perspective in the literature. Focusing on the productivity of students' prior knowledge, the analysis identified the given-ness of epsilon as an accessible and flexible resource to build on different students' justifications. The study also offers other pathways to productively reorganize students' existing knowledge about the temporal order. Findings from the study problematize the claim that there is little utility in introducing the formal definition in calculus.