Ibrahim Fatkullin

"Statistical mechanics of partitions and limit shapes of Young diagrams"

Take a positive integer, \( N \), and represent it as a sum of positive integers — it is called a partition of \( N \). In this course we will study various questions related to asymptotic properties of partitions as \( N \) tends to infinity. For example, how many partitions are there; what is a typical summand in a partition; how many summands are there on the average? Further on, each partition may be represented as a Young diagram — a special arrangement of \( N \) unit squares in a plane. Do these arrangements have what’s called a limit shape — a deterministic curve, which describes the shape of typical Young diagrams (if the latter are rescaled to have unit total area)?

Specific topics:

1. Basic combinatorial questions and ideas related to partitions of integers and sets; Young diagrams.
2. Foundations of statistical mechanics: measures on partitions; canonical and grand canonical ensembles.
4. Asymptotic properties of partitions. Limit shapes, concentration of measures in function spaces related to shapes of Young diagrams.

There is no textbook for this course; it will be based on a few papers on the subject by Vershik, Blinovskii and Yakubovich. I am finishing a paper on related topics as well.

Prerequisites: elementary combinatorics and probability; real analysis.